A New Approach to Directional Survey Interpretation and Course Correction by the Sectional Method

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An oblique circular arc representation for wellbore trajecories, a geometric analysis termed the sectional method, is presented. This approach permits projected line segments to be functions of the dogleg angle and to be related to usually measured displacements between survey stations. The advantages of this analysis are: a solution for the dogleg angle and a method of survey interpretation, the sectional method; a procedure for exact interpolation of true vertical depth, azimuth, and inclination between survey stations; a basis of solution for a computer program which provides course correction information during a turn to a target. The program provides a solution summary for a course correction from various survey stations in a well to any planned target. As a result, an optimum course correction or "minimum plugback depth" can be quickly determined. Once the desired kickoff point is selected, the program provides an exact solution of true vertical depth, azimuth, inclination, and toolface angle for every 100 of correction course length. The latter result provides a dramatic improvement in existing technology because all measurements used to control the correction run are now based on a center of turn rather than the arbitrary reference used in the typical ouija board solution; and because the solution is exact, such variables as effective toolface angle can better be evaluated and precisely corrected resulting in the smoothest possible turn with minimum doglegs.

Introduction

The radius of curvature method (Wilson, 1968) was the first method for computing wellbore trajectories using a form of circular arc approximation. Rivero (1971) presented a method for interpolation of rectangular coordinates between stations using the radius of curvature method. Taylor (1972) conclusively showed that the minimum curvature between two survey stations is given by an oblique circular arc. The widely used minimum curvature method was a result of this analysis. Zaremba (1972) used matrices and an oblique circular arc to derive an alternate analysis often referred to as the circular arc method.

Like the foregoing methods, the present analysis, termed the sectional method, also employs the oblique circular arc approximation for a segment of the wellbore trajectory. However, the sectional method differs in that the derivation approach is thought to be more easily visualized. As a result, it provides a basis of analysis for related directional surveying/drilling problems. The clarity of the method results from the projection of co-planar line segments associated with the oblique circular arc onto the horizontal and vertical planes. Thus, these easily visualized sketches show the physical significance of mathematical relationships between the dogleg angle and straight line distances in the normally referenced rectangular coordinate system. Once all the associated linear measurements are expressed as functions of the dogleg angle, the following becomes apparent: 1) a solution exists for the dogleg angle of the circular arc approximation to the wellbore trajectory between any two survey stations: and 2) Solutions for other problems (such as interpolation along the circular arc) become possible from analysis of the geometric and mathematical relationships established in development of the method.

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Concepts

The orthogonal axes shown in Fig. 1 define a three-dimensional space for the derivation of the sectional method. Point A is the origin. The positive directions are north, east and down. A "turn plane" is oriented at an oblique angle above a horizontal plane. Point S2 is a point in common with the two planes. Points, C, S1 and B, are contained in turn plane and are above the horizontal plane. The turn plane contains a "circular arc" which represents a section of the wellbore trajectory. The point, C, is the center of turn of the arc. Points, S1 and S2, represent the first and second survey stations, respectively. S1 is above S2. The angle ψ is the angle swept by the radius of the circular arc between points, S1 and S2. Point

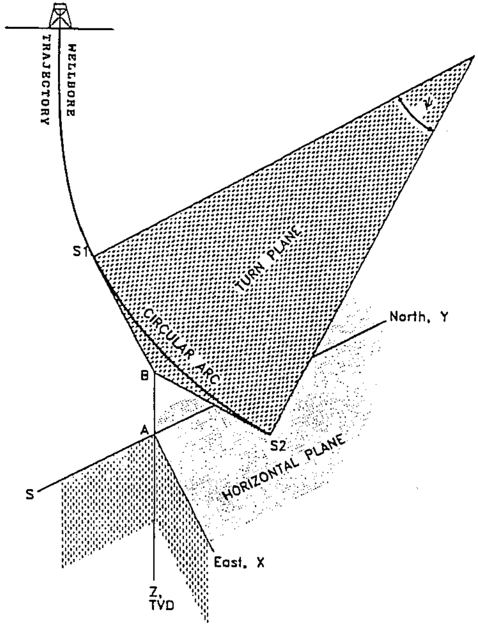

Fig. 1 Orientation defined by turn plane and circular arc approximation to a section of the wellbore

B is the point of intersection of tangents drawn to the arc at S1 and S2.

Two labeled vertical planes are shown in Fig. 2. The first plane, labeled "1st PLANE," is defined by a vertical cross section through the tangent of the arc which contains line segment S1-B. Intersection of this plane and the horizontal plane forms line "AZM1." In a similar manner, the line AZM2 is formed.

Angles, $\theta_1$and θ_2, are azimuth angles measured clockwise from north in the horizontal plane. Angle $\Delta\theta$ is the acute angle between lines AZM1 and AZM2.

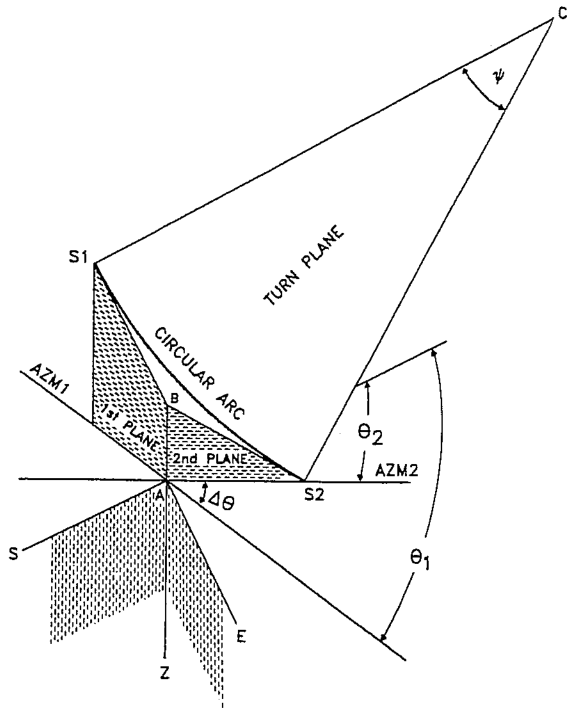

Fig. 2 Vertical planes and azimuth

Trigonometric Relationships

The Turn Plane. The "turn plane" shown in Fig. 3 contains line segments, labeled R, which represent the radius of the circular arc. Line segments $\underline{\text{S1-B}}$ and $\underline{\text{S2-B}}$ are tangents to the arc at stations S1 and S2. The length of each of these tangents is equal to the value, $R \cdot \text{Tan}(\psi/2)$. This value equals the length of the side opposite the angle, $\psi/2$, in both right

Nomenclature

D_a = magnitude of distance from chord of interpreted circular arc to arc at particular point of interest along chord or arc
Departure = horizontal distance between adjacent survey stations
DLS = dogleg severity
ΔE/W = component of departure of survey station in east/west direction (east taken as positive)
F_c = ratio used for interpolation; linear displacement (vertical, horizontal or measured depth) from first survey station to point of interest (or related point on chord of arc) divided by total length calculated or measured between two stations
F' = ratio used to take advantage of symmetry about line of bisection of dogleg angle, ψ; used for derivation of D_a and related to F_c by $F' = F_c$ for $0 \leq F_c \leq 0.5$ and $F' = 1 - F_c$ for $0.5 < F_c \leq 1.0$
F_m = ratio of measured course length between first survey station and interpolation point to total length between stations
H = course departure
MD = measured depth (course length)
ΔMD = change in measured depth (course length)
ΔN/S = component of departure of survey station in north/south direction (north taken as positive)
R = turn radius for circular arc interpreted between survey stations
S1 = shallower survey station of adjacent pair
S2 = deeper survey station of adjacent pair
TVD = true vertical depth
$\Delta X_i, \Delta Y_i, \Delta Z_i$ = coordinate displacements between first survey station, S1,and point on circular arc where interpolation of coordinates is desired
$\Delta X_t, \Delta Y_t, \Delta Z_t$ = coordinate displacements between first survey station, S1, and the second, S2
ψ = angle of turn for circular arc interpreted between survey stations; also, dogleg angle
ϕ = inclination angle
ω = angle subtended by arc drawn between first survey station and interpolation point
θ = azimuth angle
δ = angle subtended by arc drawn between interpolation point and midpoint of arc between S1 and S2; positive for $\omega \leq (\psi/2)$, otherwise negative
π = 3.14159265 (mathematical constant)
α, β, γ = direction cosines for defined line segment and associated with coordinate directions X, Y and Z, respectively

NOTE: Oilfield units understood unless otherwise specified.

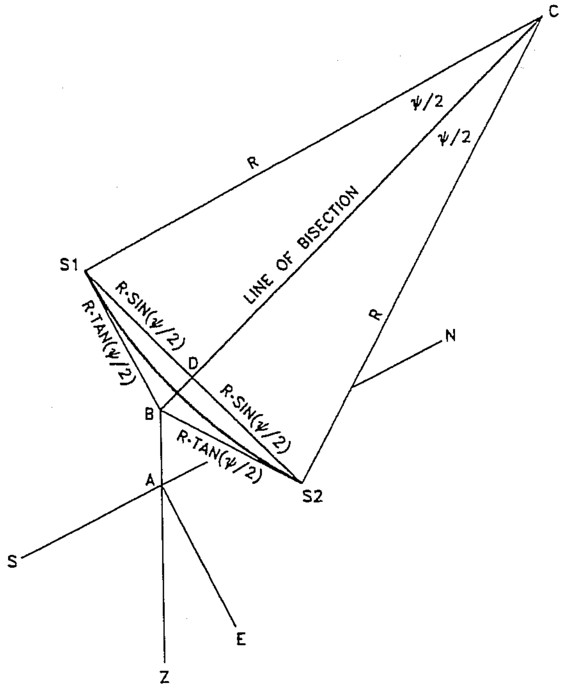

Fig. 3 Turn plane detail

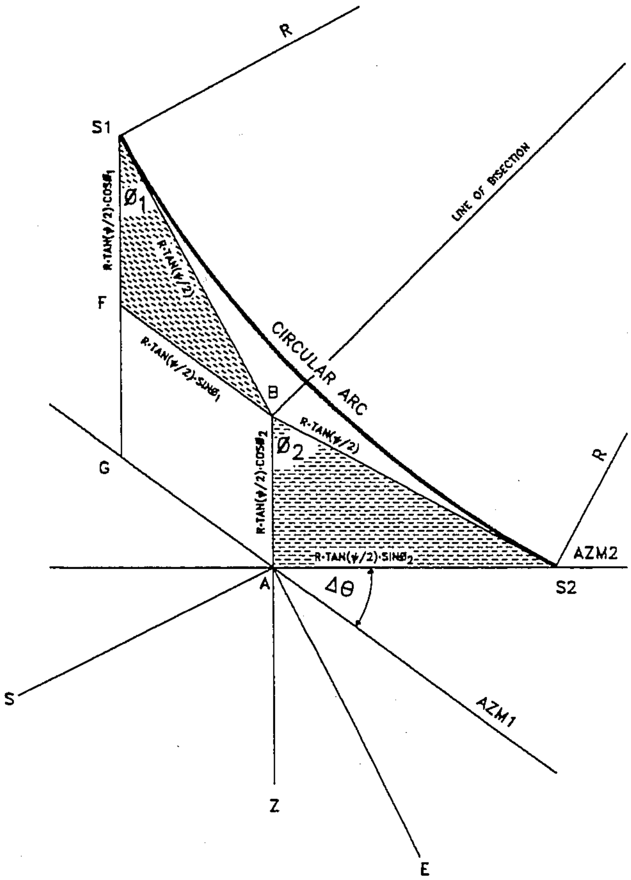

Fig. 4 Vertical plane detail

triangles, C-S1-B and C-S2-B. The line segment, B-C, labeled "line of biscetion," is the hypotenuse for both of these triangles. Line segment S1-D-S2 is the chord of the circular arc and the "straight line" distance between stations S1 and S2. Line segment B-C bisects both angle ψ and segment S1-D-S2. Line segments S1-D and S2-D are created by this bisection and

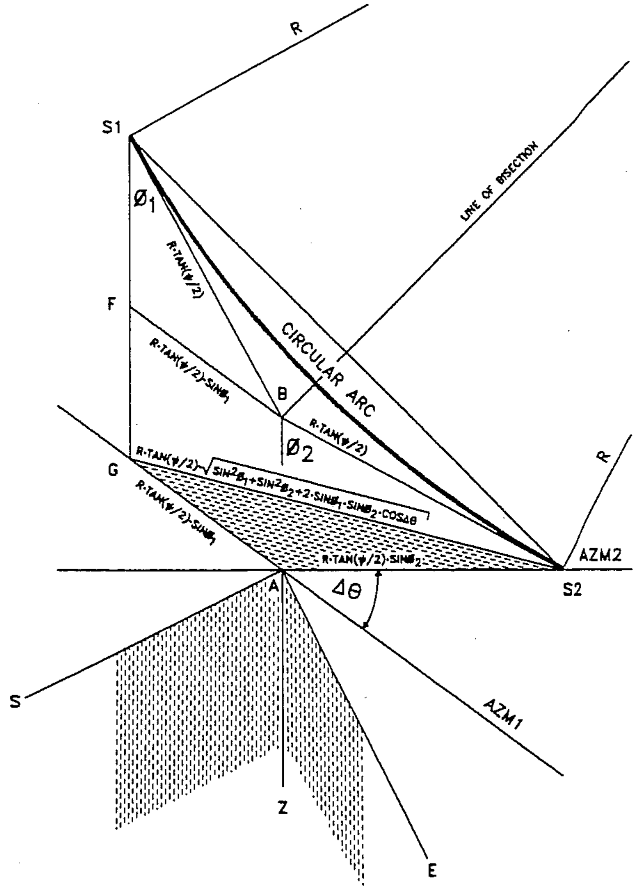

Fig. 5 Horizontal plane detail

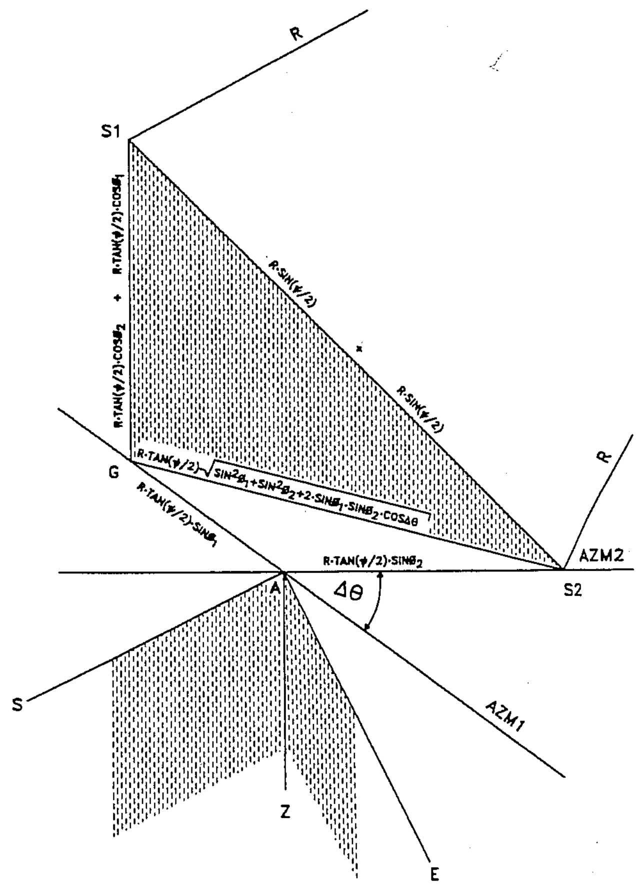

Fig. 6 Dogleg severity detail

each is part of a right triangle wherein each adjoining radius is the hypotenuse. These triangles, respectively, are C-D-S1 and C-D-S2. The lengths of line semgents S1-D and S2-D can each be expressed as $R \cdot \sin(\psi/2)$; and the "straight line" distance from S1 to S2 is $2R \cdot \sin(\psi/2)$.

Vertical Planes. Refer now to Fig. 4. Angle ϕ_1 is the inclination at the first survey station, S1. By construction, point B is directly above point A. Line segments S1-B, F-B, and G-A are contained in the same vertical plane. Segment F-B is a horizontal projection of segment S1-B and is the side opposite angle ϕ_1 in the right triangle S1-F-B. Thus, the length of F-B is $\text{S1-B} \cdot \sin\phi_1$ or, by substituting $R \cdot \tan(\psi/2)$ for S1-B, this length is $R \cdot \tan(\psi/2) \cdot \sin\phi_1$. Line segment S1-F is a vertical projection of S1-B. Its length is $R \cdot \tan(\psi/2) \cdot \cos\phi_1$.

In a similar analysis, ϕ_2 is the inclination angle at S2, the second survey station. Line segment S2-A is the side opposite angle ϕ_2 in right triangle S2-A-B. The length of this segment is $R \cdot \tan(\psi/2) \cdot \sin(\phi_2)$. The length of the vertical line segment, B-A, is $R \cdot \tan(\psi/2) \cdot \cos(\phi_2)$. It should be noted that the expression for the length of S1-G, the total change in true vertical depth between the two survey stations, is $R \cdot \tan(\psi/2) \cdot [\cos(\phi_1) + (\phi_2)]$.

The Horizontal Plane. Reference is now made to Fig. 5 where consideration is given to triangle G-A-S2. The length of the segment G-S2 in this triangle is given by the law of cosines. By substitution of $-\cos\Delta\theta$ for the cosine of angle G-A-S2, the law of cosines for the length of G-S2 is given in Appendix A, Eq. (1), as

$$\text{G-S2} = R \cdot \tan(\psi/2) \cdot \sqrt{\sin^2\phi_1 + \sin^2\phi_2 + 2 \cdot \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta} \quad (1)$$

As noted in Appendix A for the trivial case where $\psi = 0$, the tangential method (API, 1985) should be used for this "no-curvature" case. This method provides the following simple equations which produce no error:

$$\Delta N/S = \Delta MD \cdot \sin\phi_2 \cdot \cos\theta_2 \quad (14)$$

$$\Delta E/W = \Delta MD \cdot \sin\phi_2 \cdot \sin\theta_2 \quad (15)$$

$$\Delta TVD = \Delta MD \cdot \cos\phi_2 \quad (16)$$

Example Calculation

Typical application of the sectional method is shown in the forthcoming example. Calculations for displacements and dogleg severity listed in Table 1 for the measured depth interval, 900–950 ft, are detailed as follows:

Input Data for Calculation:

Given at 900 ft MD—Azimuth, $\theta_1 = 225$ deg; inclination, $\phi_1 = 10.75$ deg; measured depth, $MD_1 = 900$ft; true vertical depth, $TVD_1 = 898.40$ ft; north/south displacement, N/S_1, $= -13.47$ ft; east/west displacement, $E/W_1 = -66.70$ ft

Given at 950 ft MD—azimuth, $\theta_2 = 231$ deg; inclination, $\phi_2 = 13.50$ deg; measured depth, $MD_2 = 950$ ft

Find

1) dogleg severity between survey stations; 2) true vertical depth at the second station, TVD_2; 3) north/south displacement at second station, N/S_2; 4) east/west displacement at the second station, E/W_2.

Dogleg Severity. From Eq. (7), the dogleg severity equation is

$$DLS = (200/\Delta MD) \cdot \cos^{-1}\sqrt{1/2 \cdot (1 + \cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta)}$$

Substituting input data for the variables gives

$$DLS = (200/50) \cdot \cos - 1\sqrt{1/2 \cdot [1 + \cos(10.75) \cdot \cos(13.5) + \sin(10.75) \cdot \sin(13.5) \cdot \cos(6)]}$$

$$DLS = 6.0429 \text{ deg per } 100 \text{ ft}$$

Geometric Solution Summary

Reference is now made to Fig. 6. At this point it should be noted that expressions for all line segments in triangle G-A-S2 have been developed. Further, the relationship developed in Eq. (1) is valid for all values of $\Delta\theta$ between 0 and 180 deg. From Fig. 6 it is also apparent that another expression for line segment G-S2 is possible by means of the pythagorean theorem in the right triangle S1-G-S2. This duplicity of solution results in the number of available equations to be equal to the number of unknowns in the mathematical relationships developed in the four planes (3 vertical and 1 horizontal) related to characteristics of the circular arc between survey stations S1 and S2. As a result, the dogleg angle, ψ, and the dependent radius, R, can be solved for. All other displacements are functions of these two unknown variables and known measurements.

In summary, the resulting equations developed by the sectional method for directional survey analysis are:

$$DLS = (200/\Delta MD) \cdot \cos^{-1}\sqrt{1/2 \cdot (1 + \cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta)} \quad (7)$$

$$\Delta N/S = [(180 \cdot \Delta MD)/(\pi \cdot \psi)] \cdot \tan(\psi/1) \cdot (\sin\phi_1 \cdot \cos\theta_1 + \sin\phi_2 \cdot \cos\theta_2) \quad (10)$$

$$\Delta E/W = [(180 \cdot \Delta MD)/(\pi \cdot \psi)] \cdot \tan(\psi/2) \cdot (\sin\phi_1 \cdot \cos\theta_1 + \sin\phi_2 \cdot \cos\theta_2) \quad (11)$$

$$\Delta TVD = R \cdot \tan(\psi/2) \cdot (\cos\phi_1 + \cos\phi_2) \quad (12)$$

True Vertical Depth. From the forthcoming, the dogleg angle is

$\psi = DLS \cdot \Delta MD/100 = 3.0215$

and from Eq. (12)

$\Delta TVD = [(180 \cdot 50)/(\pi \cdot 3.0215)] \cdot \tan(3.0215/2) \cdot [\cos(10.75) + \cos(13.5)]$

$\Delta TVD = 48.88$ ft

And because $TVD_2 = TVD_1 + \Delta TVD$, then $TVD_2 = 898.40 + 48.88 = 947.28$ ft

North/South Displacement. From Eq. (10)

$$\Delta N/S = [(180 \cdot 50)/(\pi \cdot 3.0215)] \cdot \tan(3.0215/2) \cdot [\sin(10.75) \cdot \cos(225) + \sin(13.5) \cdot \cos(231)]$$

$\Delta N/S = -6.97$ ft.

And because

$N/S_2 = N/S_1 + \Delta N/S$

then $N/S_2 = -13.47 + (-6.97) = -20.44$ ft

East/West Displacement. From Eq. (11)

$\Delta E/W = [(180 \cdot 50)/(\pi \cdot 3.0215)] \cdot \tan(3.0215/2) \cdot [\sin(10.75) \cdot \sin(225) + \sin(13.5) \cdot \sin(231)]$

$\Delta E/W = -7.84$ ft

and because

$E/W_2 = E/W_1 + \Delta E/W$

then $E/W_2 = -66.7 + (-7.84) = -74.54$ ft

Table 1 Comparison of survey methods

Base Survey Data			Minimum Curvature Solution				Sectional Solution			
				RelativeCoords.		Dogleg Severity		RelativeCoords.		Dogleg Severity
INC	AZM	Measured Depth	Vertical Depth	North= + South= -	East= + West= -	Deg. per 100 Feet	Vertical Depth	North= + South= -	East= + West= -	Deg. per 100 Feet
0.00	360.00	0.00	0.00	2.56	-45.00	-	0.00	2.56	-45.00	-
0.00	360.00	50.00	50.00	2.56	-45.00	0.0000	50.00	2.56	-45.00	.0000
0.25	16.00	100.00	100.00	2.66	-44.97	0.5000	100.00	2.66	-44.97	0.5000
0.25	350.00	150.00	150.00	2.88	-44.96	0.2250	150.00	2.88	-44.96	0.2250
0.50	44.00	200.00	200.00	3.14	-44.83	0.8138	200.00	3.14	-44.83	0.8138
0.25	57.00	250.00	250.00	3.36	-44.58	0.5250	250.00	3.36	-44.58	0.5250
1.75	282.00	300.00	299.99	3.58	-45.24	3.8697	299.99	3.58	-45.24	3.8697
1.50	285.00	350.00	349.96	3.90	-46.62	0.5280	349.97	3.90	-46.62	0.5280
1.75	276.00	400.00	399.94	4.15	-48.01	0.7131	399.95	4.15	-48.01	0.7131
2.25	276.00	450.00	449.91	4.34	-49.74	1.0000	449.92	4.34	-49.74	1.0000
2.25	275.00	500.00	499.88	4.52	-51.70	0.0785	499.88	4.52	-51.70	0.0785
2.50	269.00	581.00	580.81	4.63	-55.05	0.4349	580.81	4.63	-55.05	0.4349
1.75	250.50	600.00	599.79	4.53	-55.74	5.3011	599.80	4.53	-55.74	5.3011
0.75	142.50	650.00	649.78	4.01	-56.26	4.2124	649.79	4.01	-56.26	4.2124
1.00	151.00	700.00	699.77	3.37	-55.85	0.5620	699.78	3.37	-55.85	0.5620
2.50	201.50	750.00	749.74	1.98	-56.03	4.0345	749.76	1.98	-56.03	4.0345
9.25	213.00	850.00	849.16	-6.80	-61.22	6.8183	849.18	-6.80	-61.22	6.8183
10.75	225.00	900.00	898.40	-13.47	-66.70	5.1196	898.43	-13.47	-66.70	5.1196
13.50	231.00	950.00	947.28	-20.44	-74.54	6.0429	947.31	-20.45	-74.54	6.0429
19.75	234.50	1050.00	1043.06	-37.62	-97.39	6.3270	1043.08	-37.62	-97.39	6.3270
22.25	239.00	1100.00	1089.74	-47.40	-112.38	5.9467	1089.76	-47.40	-112.38	5.9467
24.25	239.50	1150.00	1135.67	-57.49	-129.34	4.0194	1135.70	-57.49	-129.35	4.0194
26.50	234.50	1200.00	1180.85	-69.18	-147.28	6.2106	1180.88	-69.18	-147.28	6.2106
28.75	234.50	1300.00	1269.43	-96.10	-185.02	2.2500	1269.47	-96.11	-185.03	2.2500
29.00	235.00	1350.00	1313.21	-110.04	-204.74	0.6951	1313.25	-110.04	-204.74	0.6951
29.50	236.50	1400.00	1356.84	-123.78	-224.93	1.7744	1356.88	-123.79	-224.94	1.7744
29.00	237.00	1450.00	1400.46	-137.18	-245.36	1.1130	1400.50	-137.18	-245.37	1.1130
27.25	238.00	1550.00	1488.64	-162.51	-285.11	1.8123	1488.69	-162.52	-285.12	1.8123
26.25	241.00	1617.00	1548.47	-177.82	-311.07	2.5075	1548.53	-177.83	-311.09	2.5075
25.25	243.00	1663.00	1589.90	-187.21	-328.71	2.8796	1589.96	-187.22	-328.73	2.8796
21.50	243.00	1832.00	1745.00	-217.64	-388.44	2.2189	1745.06	-217.65	-388.46	2.2189
19.00	243.00	1925.00	1832.23	-232.25	-417.12	2.6882	1832.30	-232.27	-417.14	2.6882
16.75	242.00	2019.00	1921.68	-245.56	-442.71	2.4157	1921.76	-245.57	-442.73	2.4157
14.75	242.00	2114.00	2013.10	-257.67	-465.47	2.1053	2013.19	-257.68	-465.50	2.1053
13.50	243.00	2209.00	2105.22	-268.38	-486.03	1.3406	2105.32	-268.39	-486.06	1.3406
12.75	245.00	2302.00	2195.79	-277.64	-505.01	0.9427	2195.89	-277.66	-505.03	0.9427
12.00	246.00	2395.00	2286.63	-285.91	-523.14	0.8387	2286.73	-285.93	-523.16	0.8387
10.50	243.00	2430.00	2320.95	-288.84	-529.30	4.5990	2321.05	-288.86	-529.33	4.5990
12.00	241.00	2523.00	2412.16	-297.37	-545.31	1.6663	2412.26	-297.39	-545.34	1.6663
11.75	238.00	2685.00	2570.69	-314.28	-574.03	0.4111	2570.80	-314.30	-574.06	0.4111
11.75	237.00	2727.00	2611.81	-318.87	-581.24	0.4849	2611.92	-318.89	-581.27	0.4849
10.75	233.00	2791.00	2674.58	-326.02	-591.47	1.9811	2674.69	-326.03	-591.50	1.9811
11.25	233.00	2852.00	2734.46	-333.02	-600.77	0.8197	2734.57	-333.04	-600.80	0.8197
10.75	233.00	2914.00	2795.32	-340.14	-610.22	0.8065	2795.43	-340.16	-610.25	0.8065
10.25	232.00	2983.00	2863.16	-347.79	-620.19	0.7712	2863.27	-347.81	-620.22	0.7712
8.75	232.00	3068.00	2946.99	-356.43	-631.25	1.7647	2947.10	-356.45	-631.28	1.7647
8.00	228.00	3159.00	3037.02	-364.93	-641.41	1.0432	3037.13	-364.94	-641.44	1.0432
7.50	230.00	3230.00	3107.37	-371.21	-648.63	0.8000	3107.48	-371.23	-648.66	0.8000
7.00	228.00	3319.00	3195.65	-378.57	-657.11	0.6292	3195.77	-378.59	-657.14	0.6292
7.00	230.00	3410.00	3285.98	-385.85	-665.48	0.2678	3286.10	-385.87	-665.51	0.2678
7.50	228.00	3503.00	3378.23	-393.55	-674.33	0.6022	3378.35	-393.57	-674.36	0.6022
7.75	229.00	3592.00	3466.44	-401.38	-683.17	0.3180	3466.56	-401.39	-683.20	0.3180
8.00	230.00	3663.00	3536.77	-407.69	-690.57	0.4015	3536.89	-407.71	-690.60	0.4015

Comparison of Methods

For comparison, the sectional method was applied to data published in Taylor's paper (1972). Table 1 is a summary of his data and computations. It may be noted that computations with the two methods are almost equal. All dogleg severities agree to four decimal places. Bottomhole location agrees to within 0.04 ft. The minor differences occur because Taylor assumed a straight line rather than an arc for small values of dogleg angle.

API Bulletin D20 (1985) contains three equations for dogleg severity in addition to Taylors. Table 2 shows computed DLS values for the foregoing equations and the sectional method. It may be noted that no significant differences exist between any of the methods except the questionable results from the modified radius of curvature method.

Interpolation Between Stations

Closer analysis of the geometric and mathematical relationships established during development of the sectional method allows one to recognize that it would be possible to develop a method for interpolation between stations for the section of

Table 2 Comparison of common equations for dogleg severity

Survey Data Base			Dogleg Severity Solutions From API Bulletin D20 (Min. Curvature DLS Values From TABLE 1 - Base For % Diff. Calculation)							
			Sectional Method		Radius of Curvature Method		Lubinski's DLS Solution		Wilson's DLS for Tangential Method	
INC	AZM	Measured Depth	DLS	% Diff.	DLS	% Diff.	DLS	% Diff.	DLS	% Diff.
0.00	360.00	0.00	-	-	-	-	-	-	-	-
0.00	360.00	50.00	.0000	0.0	0.0000	0.0	0.0000	0.0	.0000	0.0
0.25	16.00	100.00	0.5000	.0	0.5191	3.8	0.5000	.0	0.5000	.0
0.25	350.00	150.00	0.2250	.0	0.2269	0.8	0.2250	.0	0.2250	.0
0.50	44.00	200.00	0.8138	.0	1.0669	31.1	0.8138	.0	0.8138	.0
0.25	57.00	250.00	0.5250	.0	0.5127	2.3	0.5250	.0	0.5250	.0
1.75	282.00	300.00	3.8697	.0	8.7742	126.7	3.8697	.0	3.8697	.0
1.50	285.00	350.00	0.5280	.0	0.5241	0.7	0.5280	.0	0.5280	.0
1.75	276.00	400.00	0.7131	.0	0.7431	4.2	0.7131	.0	0.7131	.0
2.25	276.00	450.00	1.0000	.0	1.0000	.0	1.0000	.0	1.0000	.0
2.25	275.00	500.00	0.0785	.0	0.0785	.0	0.0785	.0	0.0785	.0
2.50	269.00	581.00	0.4349	.0	0.4468	2.7	0.4349	.0	0.4349	.0
1.75	250.50	600.00	5.3011	.0	4.9420	6.8	5.3011	.0	5.3011	.0
0.75	142.50	650.00	4.2124	.0	3.4632	17.8	4.2124	.0	4.2124	.0
1.00	151.00	700.00	0.5620	.0	0.5814	3.5	0.5620	.0	0.5620	.0
2.50	201.50	750.00	4.0345	.0	5.3300	32.1	4.0345	.0	4.0345	.0
9.25	213.00	850.00	6.8183	.0	6.9985	2.6	6.8183	.0	6.8183	.0
10.75	225.00	900.00	5.1196	.0	5.3889	5.3	5.1196	.0	5.1196	.0
13.50	231.00	950.00	6.0429	.0	6.1723	2.1	6.0429	.0	6.0429	.0
19.75	234.50	1050.00	6.3270	.0	6.3609	0.5	6.3270	.0	6.3270	.0
22.25	239.00	1100.00	5.9467	.0	6.0509	1.8	5.9467	.0	5.9467	.0
24.25	239.50	1150.00	4.0194	.0	4.0210	.0	4.0194	.0	4.0194	.0
26.50	234.50	1200.00	6.2106	.0	6.3371	2.0	6.2106	.0	6.2106	.0
28.75	234.50	1300.00	2.2500	.0	2.2500	0.0	2.2500	.0	2.2500	.0
29.00	235.00	1350.00	0.6951	.0	0.6964	0.2	0.6951	.0	0.6951	.0
29.50	236.50	1400.00	1.7744	.0	1.7839	0.5	1.7744	.0	1.7744	.0
29.00	237.00	1450.00	1.1130	.0	1.1113	0.2	1.1130	.0	1.1130	.0
27.25	238.00	1550.00	1.8123	.0	1.8089	0.2	1.8123	.0	1.8123	.0
26.25	241.00	1617.00	2.5075	.0	2.4798	1.1	2.5075	.0	2.5075	.0
25.25	243.00	1663.00	2.8796	.0	2.8576	0.8	2.8796	.0	2.8796	.0
21.50	243.00	1832.00	2.2189	.0	2.2189	.0	2.2189	.0	2.2189	.0
19.00	243.00	1925.00	2.6882	.0	2.6882	.0	2.6882	.0	2.6882	.0
16.75	242.00	2019.00	2.4157	.0	2.4132	0.1	2.4157	.0	2.4157	.0
14.75	242.00	2114.00	2.1053	.0	2.1053	.0	2.1053	.0	2.1053	.0
13.50	243.00	2209.00	1.3406	.0	1.3385	0.2	1.3406	.0	1.3406	.0
12.75	245.00	2302.00	0.9427	.0	0.9357	0.7	0.9427	.0	0.9427	.0
12.00	246.00	2395.00	0.8387	.0	0.8369	0.2	0.8387	.0	0.8387	.0
10.50	243.00	2430.00	4.5990	.0	4.5615	0.8	4.5990	.0	4.5990	.0
12.00	241.00	2523.00	1.6663	.0	1.6737	0.4	1.6663	.0	1.6663	.0
11.75	238.00	2685.00	0.4111	.0	0.4075	0.9	0.4111	.0	0.4111	.0
11.75	237.00	2727.00	0.4849	.0	0.4849	.0	0.4849	.0	0.4849	.0
10.75	233.00	2791.00	1.9811	.0	1.9495	1.6	1.9811	.0	1.9811	.0
11.25	233.00	2852.00	0.8197	.0	0.8197	.0	0.8197	.0	0.8197	.0
10.75	233.00	2914.00	0.8065	.0	0.8065	.0	0.8065	.0	0.8065	.0
10.25	232.00	2983.00	0.7712	.0	0.7692	0.3	0.7712	.0	0.7712	.0
8.75	232.00	3068.00	1.7647	.0	1.7647	.0	1.7647	.0	1.7647	.0
8.00	228.00	3159.00	1.0432	.0	1.0264	1.6	1.0432	.0	1.0432	.0
7.50	230.00	3230.00	0.8000	.0	0.7944	0.7	0.8000	.0	0.8000	.0
7.00	228.00	3319.00	0.6292	.0	0.6250	0.7	0.6292	.0	0.6292	.0
7.00	230.00	3410.00	0.2678	.0	0.2678	.0	0.2678	.0	0.2678	.0
7.50	228.00	3503.00	0.6022	.0	0.6065	0.7	0.6022	.0	0.6022	.0
7.75	229.00	3592.00	0.3180	.0	0.3192	0.4	0.3180	.0	0.3180	.0
8.00	230.00	3663.00	0.4015	.0	0.4030	0.4	0.4015	.0	0.4015	.0

the wellbore trajectory represented by the circular arc. Figures 8 and 9 are used for this analysis. It should be observed from Fig. 8 that for any depth of interest, point K, along the segment of the wellbore trajectory represented by the circular arc between stations S1 and S2, the location of that depth could be determined if one could calculate the distance and direction along the chord of the arc to point I and then vectorially add the directed segment, D_a, normal to the chord and extending to point K on the arc. This is, in fact, what is developed in Appendixes B through D. Figure 9 is used to develop a simple method to provide two additional solutions: 1) The distance along the chord to point I for some known distance (change in measured depth) from the first survey station to point K (assumed point of interest); 2) The magnitude of the distance from point I to the arc at point K. The required directions for the displacement vectors from the first survey station to point K are then derived from previously developed expressions for the line segments shown in Fig. 8. Once the precise location of point K is known, the interpolated inclination and azimuth can be solved for from existing equations established in the

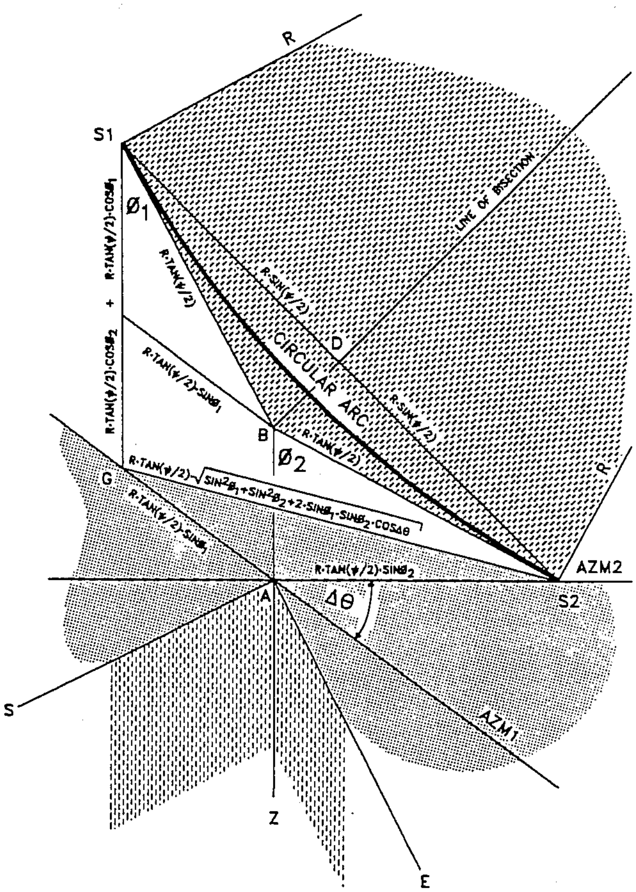

Fig. 7 Displacement detail

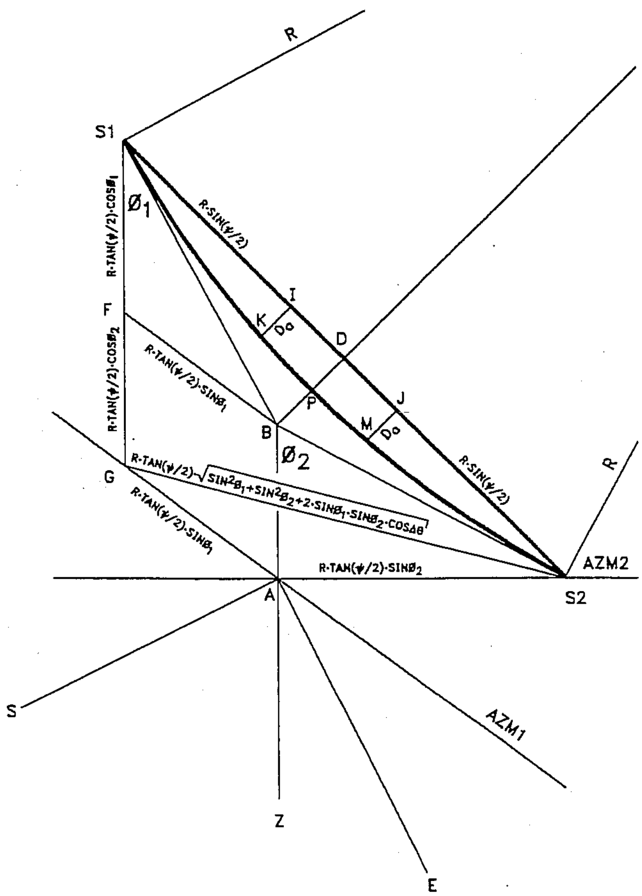

Fig. 8 Arc/chord overview

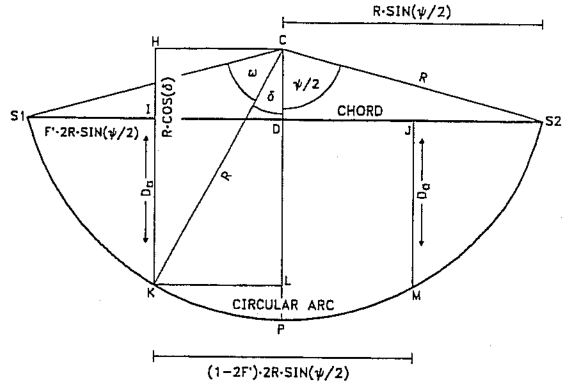

Fig. 9 Arc/chord detail

development of the sectional method. Although the interpolation process developed in the Appendixes is slightly lengthy for hand calculation, it is very straightforward and lends itself toward simple programming on any computer or programmable calculator. The stepwise process would be as follows:

1 Given the depth of interest, calculate F_m, the fractional distance that it occurs between the two adjacent survey stations, $(MD_i - MD_1)/(MD_2 - MD_1)$.

2 Calculate the fractional distance along the chord, F_c, associated with the foregoing fractional distance along the interpreted circular arc. This is given in Eq. (27) as

$$F_c = 0.5 \cdot \{1 - \sin[\psi \cdot (0.5 - F_m)]/\sin(\psi/2)\}$$

Note that $\psi/2$ was already solved for in calculation of the dogleg angle between the two stations.

3 The displacement distance from the chord to the arc (D_a in Fig. 8) is given in Eq. (32) as

$$D_a = [(180 \cdot \Delta MD)/(\pi \cdot \psi)] \cdot \{\cos[\sin^{-1}[(1-2F_c) \cdot \sin(\psi/2)]] - \cos(\psi/2)\}$$

4 From Eq. (95), solve for the variable ρ

$$\rho = \sqrt{2 \cdot [1 - \sin(\phi_1)\sin(\phi_2)\cos|\Delta\theta| - \cos(\phi_1)\cos(\phi_2)]}$$

5 The direction cosines for the displacement vector, D_a, are solved for by Eqs. (46)–(48) as follows:

$$\alpha = [\sin(\phi_1)\sin(\theta_1) - \sin(\phi_2)\sin(\theta_2)]/\rho$$

$$\beta = [\sin(\phi_1)\cos(\theta_1) - \sin(\phi_2)\cos(\theta_2)]/\rho$$

$$\gamma = [\cos(\phi_1) - \cos(\phi_2)]/\rho$$

6 Given the known displacements between stations S1 and S2 (ΔX_t, ΔY_t, and ΔX_t), the displacements between station S1 and the depth of interest, assumed to be on the circular arc at point K in Fig. 8, can be calculated by Eq. (49)–(51) as follows:

$$\Delta X_i = F_c \cdot \Delta X_t + D_a \cdot \alpha$$

$$\Delta Y_i = F_c \cdot \Delta Y_t + D_a \cdot \beta$$

$$\Delta Z_i = F_c \cdot \Delta Z_t + D_a \cdot \gamma$$

7 The turn angle from station S1 to point K has the same rate of turn (dogleg severity) as the dogleg angle between stations S1 and S2. Thus, the turn angle to the depth of interest can be calculated as follows:

$$\psi_i = F_m \cdot \psi$$

The inclination at the interpolated survey station at this point can be calculated from Eq. (52) as

$$\phi_i = \cos^{-1}\{[\Delta Z_i/R \cdot \tan(\psi_i/2)] - \cos(\phi_1)\}$$

8 By Eq. (53), the change in azimuth between station S1 and the interpolated survey station on the circular arc is

Table 3 Interpolation of survey data by the sectional method

Survey Data Interpolated ( )			Interpolated Data				Sectional Solution			
INC	AZM	Measured Depth	Vertical Depth	Rel. Coordinates North= + South= -	Rel. Coordinates East= + West= -	Dogleg Severity Deg. per 100 Feet	Vertical Depth	Rel. Coordinates North= + South= -	Rel. Coordinates East= + West= -	Dogleg Severity Deg. per 100 Feet
2.50	201.50	750.00	749.74	1.98000	-56.0300	-	749.74	1.98	-56.03	4.0345
(3.16)	(204.84)	760.00	759.73	1.52680	-56.2258	6.8183				
(3.83)	(207.02)	770.00	769.71	0.97893	-56.4934	6.8183				
(4.50)	(208.55)	780.00	779.68	0.33646	-56.8328	6.8183				
(5.18)	(209.68)	790.00	789.65	-0.40050	-57.2439	6.8183				
(5.86)	(210.55)	800.00	799.60	-1.23187	-57.7266	6.8183				
(6.53)	(211.24)	810.00	809.54	-2.15751	-58.2810	6.8183				
(7.21)	(211.81)	820.00	819.47	-3.17731	-58.9069	6.8183				
(7.89)	(212.27)	830.00	829.38	-4.29111	-59.6042	6.8183				
(8.57)	(212.66)	840.00	839.28	-5.49876	-60.3729	6.8183				
9.25	213.00	850.00	849.16	-6.80008	-61.2128	6.8183	849.16	-6.80	-61.21	6.8183

$$\Delta\theta = \cos^{-1}\{[2\cdot\cos(\psi_i/2) - 1 - \cos(\phi_1)\cos(\phi_i)]/[\sin(\phi_1)\sin(\phi_i)]\}$$

9 The direction of azimuth change is calculated by

$$(\theta_2 - \theta_1)/\Delta\theta$$

Then by Eq. (54), the azimuth at the second station is

$$\theta_1' = \theta_1 \pm \Delta\theta$$

Table 3 shows interpolated coordinates, as well as inclinations and azimuth, between the measured depth of 750 and 850 ft. Note that DLS stays constant in value, as it should.

Application of Concepts

Derivation of the interpolation process is simply one example of how the geometric and mathematical relationships developed in the sectional method can be extended to solutions for others directional drilling problems. The method, is fact, provides a basis of definition and basic building blocks for sound development of other directional drilling solutions. Another such extension of the basic circular arc solution provided in the sectional method is shown in Table 4. This is a listing of output data from a computer program designed for planning and monitoring a course correction to a new target in a directional well. The program is basically divided into two sections. Section 1 provides a quick look for the overall design as various stations are selected sequentially from some lowest anticipated kickoff depth of some station uphole. The station selected for kickoff is considered satisfactory for the dogleg severity considered allowable and the overall dimensions of the correction plan (correction run length, formation penetrated during the correction run, target entry angle, etc.). Part 1 in Table 4 shows the section where the target coordinates are entered and successive stations uphole are entered under "Present Position" as the situation at each station of interest is evaluated. Part 1A shows the overall dimensions of the course corrected. The minimum dogleg severity to just hit the target is listed first. This provides a quick-glance, relative estimate of how far to move uphole if the initial estimated kickoff point is too close to the target. This process is continued until a depth is zeroed in where correction run length, target entry angle, TVD and displacement at the end of the correction run, and straight section length into the target are satisfactory.

Once the kickoff station is selected, the program proceeds to calculate a survey schedule for every 100 ft of the correction run. The output includes all displacements, azimuth, inclination and effective toolface angle at each station. Verification is provided by including the coordinates for the center of turn. Distance from each station to the center of the turn should be equal to the turn radius and the toolface should always be pointed toward the same center. Note that as the station TVD drops below the center of turn (500–600 ft) the effective toolface angle swings toward the top of the hole (angle less than 90 deg). Further indication that the turn is progressing normally is based on a geologic analogy. From geology, the strike of a plane is the direction along which the dip angle is least. In this plane of turn, the same phenomenon occurs. Inclination drops to a minimum as azimuth swings between 217 and 211 deg and then begins to increase. This sequence of azimuth and inclination change is difficult to visualize or predict without knowing something about the plane of turn. It should be noted that this turn could not be predicted with the standard ouija board approach.

As a final check of the plan (see "VERIFICATION"), the calculated azimuths and inclinations were entered into the directional worksheet as though the measurements had been taken. The resulting check on displacements and calculated DLS verify the accuracy of the correction plan. The program can easily be rerun at each station to get an updated plan each time allowing for a smooth correction from errors caused by deviations of the actual drilling progress from the planned course.

Conclusions

1 The graphical analysis used in the development of the sectional method has resulted in a set of transformation equations for the calculation of wellbore tracjectories and dogleg severities. Utilization of sectional method procedures and equations has been demonstrated to provide reliable results.

2 An exact set of equations for determination of ΔX, ΔY, ΔZ, ϕ and θ, have been developed for intermediate points between survey stations. Definitions and relationships developed within the model for the sectional method are required for derivation of these equations. Hence, they are an extension of the sectional method and an added benefit resulting from this type of analysis for a circular arc.

3 As a result of development of the sectional method, all linear projections associated with a circular arc have been defined as a function of the calculated dogleg angle and/or measured variables. Thus, the sectional method provides a based of definitions, not previously defined in other methods, for directional analysis and application of a circular arc model.

4 The potential for application of the concepts and definitions, defined herein, to analysis of related directional problems was demonstrated by the interpolation method and the computer solution for the turn to a new target. In the latter

Table 4

PART 1

NEW TARGET DIRECTION PROGRAM

```
Present Position:
                1) East/West (+/-) Rel. Coord.: -1000
                2) North/South (+/-) Rel. Coord.: -500
                3) True Vertical Depth: 3500
                4) Inclination (Degs): 35
                5) Azimuth (Degs): 250
New Target Data:
                6) East/West (+/-) Rel. Coord.: -2000
                7) North/South (+/-) Rel. Coord.: -2000
                8) True Vertical Depth: 6500
                9) Desired Turn Rate (Deg/100 Ft): 3
```

PART 1A

```
Minimum DLS required to hit the target = 1.116 deg/100ft
Formation entry angle = 31.26 deg
Correction Run Length =  741.38 ft  ---> Straight Section = 2774.21 ft
Situation at end of correction run: - - - - - - - - - - - - - - - - - - - - -
       Azimuth to TGT = 209.22 deg        Inclination to TGT = 31.26 deg
      East/West (+/-) = -1297.4 ft        North/South (+/-) =  -743.7 ft
  True Vertical Depth =  4128.5 ft
```

PART 2

COURSE CORRECTION SURVEY SCHEDULE

```
Verification: Turn Radius =  1909.86 feet
              Arc Center:  X=   239.38 feet Y=  -1868.83 feet
                           Z=  3987.67 feet
```

Course Length	East/ West	North/ South	TVD	AZM	INC	TFA (LT)
0	-1000.00	-500.00	3500.00	250.0000	35.0000	116.43
100	-1052.18	-521.48	3582.55	245.1617	33.7577	112.44
200	-1100.81	-546.66	3666.20	240.0270	32.7164	108.15
300	-1145.77	-575.46	3750.74	234.6257	31.8956	103.58
400	-1186.94	-607.81	3835.93	229.0094	31.3130	98.79
500	-1224.19	-643.61	3921.53	223.2429	30.9821	93.86
600	-1257.44	-682.77	4007.31	217.4104	30.9110	88.85
700	-1286.59	-725.18	4093.04	211.5969	31.1016	83.87
741	-1297.42	-743.66	4128.45	209.2150	31.2563	81.83

VERIFICATION

Directional Drilling Worksheet (Sectional Method)

```
                                                                            Directional Program Plan:
 Company:            Rerference:                Target Coords:                      KOP:     |EOBU-TVD:
   Field:             Latitude        0         N/S= +/- : -2000.00             BU RATE:     | EOBU-MD:
    Well:            Longitude:       0         E/W= +/- : -2000.00             DO RATE:     |EOBU-DEP:
Engineer:            Well Offset:                    TVD :    6500            Entry Ang:     | 8DO-TVD:
  Survey:            N/S= +/- : -500.00          DIST-TGT:  1802.78           (-1 = N/A)     |  BDO-MD:
    Date:            E/W= +/- :-1000.00          CNTR-TGT:   225.00 Deg.      Slant Ang:     | 8DP-DEP:
                                                  "    " :  2828.43 Ft.
```

Survey Number	Measured. Depth	INC	AZM	True Vertical Depth	Relative Coords. North= + South= -	Relative Coords. East= + West= -	Relative Closure	Closure Azimuth	3-Dim Distance to Target	AZM to TGT	Inclin. to Target	Dogleg Severity Deg/100Ft
0	0	35.00	250.00	3500.00	-500.00	-1000.00	1118.03	243.43	3500.00	213.69	31.00	-
1	100	33.76	245.16	3582.55	-521.49	-1052.18	1174.32	243.64	3405.28	212.66	31.05	3.00
2	200	32.72	240.03	3666.20	-546.67	-1100.82	1229.08	243.59	3309.25	211.75	31.09	3.00
3	300	31.90	234.63	3750.73	-575.47	-1145.78	1282.18	243.33	3212.08	210.95	31.14	3.00
4	400	31.31	229.01	3835.92	-607.81	-1186.95	1333.53	242.88	3113.93	210.29	31.18	3.00
5	500	30.98	223.24	3921.52	-643.61	-1224.20	1383.08	242.27	3014.99	209.77	31.22	3.00
6	600	30.91	217.41	4007.31	-682.77	-1257.45	1430.86	241.50	2915.47	209.41	31.24	3.00
7	700	31.10	211.60	4093.04	-725.18	-1286.59	1476.89	240.59	2815.59	209.23	31.25	3.00
8	741.38	31.26	209.22	4128.45	-743.65	-1297.43	1495.44	240.18	2774.22	209.21	31.26	3.00

case, the circular arc is simply defined to contain multiple stations instead of two. The dogleg angle and center of turn now relate to the entire correction run and the known variables change. The power of the program is demonstrated in the fact that it is capable of providing not only the overall course design, but explicit measurements which can be used to accomplish the design and monitor progress. Consequently, the magnitude of the course corrections can be minimized resulting in minimum difference between planned and actual dogleg severity throughout the correction run.

References

API Bulletin D20: Directional Drilling Survey Calculation Methods and Terminology, API Production Department, Dallas, Tex., Dec., 1985.

Callas, N. P., et al., 1979, "Directional Survey Calculation Methods Compared and Programmed," *Oil and Gas Journal*, January 22, pp. 53–58.

Lubinski, A., 1957, "How to Determine Hole Curvature," *The Petroleum Engineer*, Feb., pp. B42–B47.

Lubinski, A., 1961, "Maximum Permissible Dog-legs in Rotary Boreholes," *Journal of Petroleum Technology*, Feb., pp. 175–194.

McMillian, W. H., 1981, "Planning the Directional Well—A Calculational Method," *Journal of Petroleum Technical*, June, pp. 952–962.

Rivero, J. T., 1971, "Use of the Curvature Method to Determine True Vertical Reservoir Thickness," *Journal of Petroleum Technology*, June, pp. 491–496.

Taylor, H. L., and Mason, C. M., 1972, "A Systematic Approach to Well Surveying Calculations," *Society of Petroleum Engineering Journal*, Dec., pp. 474–88.

Thorogood, J. L., 1986, "How to Get the Best Results from Well Surveying Data," *World Oil*, Apr., pp. 98–106.

Walstrom, J. E., Harvey, R. P., and Eddy, H. D., 1972, "A Comparison of Various Directional Survey Models and An Approach to Model Error Analysis," *Journal of Petroleum Technology*, Aug., pp. 935–943.

Wilson, G. J., 1968, "An Improved Method for Computing Directional Surveys," *Journal of Petroleum Technology*, Aug., pp. 871–876.

Zaremba, W. A., 1973, "Directional Survey by the Circular Arc Method," *Society of Petroleum Engineering Journal*, Feb., pp. 4–11.

APPENDIX A

Derivation of Sectional Method Equations

Dogleg Severity. Refer to Fig. 6. The length of line segment G-S2 in triangle G-A-S2 is given by the law of cosines. By substitution of $-\cos\Delta\theta$ for the cosine of angle G-A-S2, the law of cosines for the length of G-S2 is written

$$\underline{G\text{-}S2} = R \cdot \tan(\psi/2) \cdot \sqrt{\sin^2\phi_1 + \sin^2\phi_2 + 2 \cdot \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta} \tag{1}$$

Expressions now exist for the sides of the right triangle, S1-G-S2. The equation for angle ψ is developed in the following: Substitution of expressions for sides of the right triangle S1-G-S2 into the pythagorean theorem yields

$$[2R \cdot \sin(\psi/2)]^2 = [R \cdot \tan(\psi/2) \cdot (\cos(\phi_1) + \cos(\phi_2))]^2 + [R \cdot \tan(\psi/2) \cdot \sqrt{\sin^2\phi_1 + \sin^2\phi_2 + 2 \cdot \sin\phi_1 \cdot \sin\phi_1 2 \cdot \cos\Delta\theta}]^2 \tag{2}$$

Substitution of $\sin(\psi/2)/\cos(\psi/2)$ for $\tan(\psi/2)$ and division of both sides by $[R \cdot \sin(\psi/2)]^2$ yields

$$4 = [(\cos\phi_1 + \cos\phi_2)/(\cos(\psi/2)]^2 + [\sqrt{\sin^2\phi_1 + \sin^2\phi_2 + 2 \cdot \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta}/\cos(\psi/2)]^2 \tag{3}$$

Simplification and solution for $\cos(\psi/2)$ results in

$$\cos(\psi/2) = \sqrt{1/4 \cdot [(\cos\phi_1 + \cos\phi_2)^2 + \sin^2\phi_1 + \sin^2\phi_2 + 2 \cdot \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta]} \tag{4}$$

Expansion of the term, $(\cos\phi_1 + \cos\phi_2)^2$, substitution of the equality, $\sin^2\phi + \cos^2\phi = 1$, and collection of terms gives

$$\cos(\psi/2) = \sqrt{1/2 \cdot (1 + \cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta)} \tag{5}$$

Solving for ψ results in

$$\psi = 2 \cdot \cos^{-1}\sqrt{1/2 \cdot (1 + \cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta)} \tag{6}$$

Dogleg severity equals the dogleg angle, ϕ, divided by arc length expressed in hundreds of feet. Arc length is assumed here to be equal to the difference in measured depth, ΔMD, between survey stations. In oilfield units, the equation is

$$\mathrm{DLS} = (200/\Delta\mathrm{MD}) \cdot \cos^{-1}\sqrt{1/2 \cdot (1 + \cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \cdot \sin\phi_2 \cdot \cos\Delta\theta)} \tag{7}$$

Displacements. Refer to Fig. 7. From this figure, it should be observed that the north/south displacement of survey station S2 from survey station S1 is expressed

$$\Delta N/S = (\underline{G\text{-}A} \cdot \cos\theta_1) + (\text{A-S2} \cdot \cos\theta_2) \tag{8}$$

Substitution of expressions for the line segments in Eq. (8) yields

$$\begin{aligned} \Delta N/S &= [R \cdot \tan(\psi/2) \cdot \sin\phi_1] \cdot \cos\theta_1 \\ &\quad + [R \cdot \tan(\psi/2) \cdot \sin\phi_2] \cdot \cos\theta_2 \\ &= R \cdot \tan(\psi/2) \cdot (\sin\phi_1 \cdot \cos\theta_1 + \sin\phi_2 \cdot \cos\theta_2) \end{aligned} \tag{9}$$

In terms of known variables and in oilfield units, the radius of a circular arc is $(180 \cdot \Delta\mathrm{MD})/(\pi \cdot \psi)$. substitution of this expression for R in Eq. (9) gives

$$\Delta N/S = [(180 \cdot \Delta\mathrm{MD})/(\pi \cdot \psi)] \cdot \tan(\psi/2) \cdot (\sin\phi_1 \cdot \cos\theta_1 + \sin\phi_2 \cdot \cos\theta_2) \tag{10}$$

In a similar manner, the equation for the east/west displacement can be expressed

$$\Delta E/W = [(180 \cdot \Delta\mathrm{MD})/(\pi \cdot \psi)] \cdot \tan(\psi/2) \cdot (\sin\phi_1 \cdot \sin\theta_1 + \sin\phi_2 \cdot \sin\theta^2) \tag{11}$$

It was demonstrated in the "Vertical Planes" section that total change in true vertical depth, TVD, can be expressed

$$\Delta\mathrm{TVD} = R \cdot \tan(\psi/2) \cdot (\cos\phi_1 + \cos\phi_2) \tag{12}$$

Again, substituting known variables for R, the change in true vertical depth is

$$\Delta\mathrm{TVD} = [(180 \cdot \Delta\mathrm{MD})/(\pi \cdot \psi)] \cdot \tan(\psi/2) \cdot (\cos\phi_1 + \cos\phi_2) \tag{13}$$

It should be noted that if ϕ_1 is equal in value to ϕ_2 and θ_1 is equal in value to θ_2, then the dogleg angle, ψ, is zero. Division by zero in Eqs. (10), (11) and (13) for this case makes them invalid. It is recognized, however, that the tangential method (API, 1985) provides the following equations which produce no error for this "no-curvature" case:

$$\Delta N/S = \Delta\mathrm{MD} \cdot \sin\phi_2 \cdot \cos\theta_2 \tag{14}$$

$$\Delta E/W = \Delta\mathrm{MD} \cdot \sin\phi_2 \cdot \sin\theta_2 \tag{15}$$

$$\Delta\mathrm{TVD} = \Delta\mathrm{MD} \cdot \cos\phi_2 \tag{16}$$

APPENDIX B

Arc-Chord Correlations

Derivation for the Interpolation Ratio, F_c. Refer to Fig. 9. Point K is on the arc at the measured depth at which ΔX, ΔY, ΔZ, ϕ and θ are desired. Let the change in measured depth between S1 and K, divided by the total change in measured depth between S1 and S2 be F_m. Let the distance along the chord between S1 and I divided by the total chord length, between S1 and S2 be F_c. F_c is equal to 0.25 in Fig. 9. Consequently, the chord is divided into 4 line segments of equal length, S1-I, I-D, J-D and J-S2.

Define F' as

$$F' = F_c \text{ for } 0 \le F_c \le 0.5;$$

$$= 1 - F_c \text{ for } 0.5 < F_c \le 1.0 \quad (17)$$

Line segment $\underline{S1\text{-}I}$ is equal to $\underline{J\text{-}S2}$ and can be expressed

$$\underline{S1\text{-}I} = \underline{J\text{-}S2} = F' \cdot R \cdot \sin(\psi/2) \quad (18)$$

The sum of the "inner" two chord segments can be expressed as the difference between the sum of the "outer" chord segments and the total chord length, $2R \cdot \sin(\psi/2)$, as follows:

$$\underline{I\text{-}D} + \underline{D\text{-}J} = (1 - 2F') \cdot 2R \cdot \sin(\psi/2) \quad (19)$$

The angle subtended by the arc between S1 and K is the angle, ω. Therefore, the ratio, F_m can be expressed as

$$F_m = \omega/\psi \quad (20)$$

Angle, δ, is the angle subtended by the arc drawn between K and P (positive for $\omega \le (\psi/2)$, negative otherwise). The length of line segment $\underline{K\text{-}L}$ is

$$\underline{K\text{-}L} = R \cdot \sin(\delta) \quad (21)$$

Because $\underline{K\text{-}L}$ is also equal to half the sum of the "inner" chord segments, the following can be written:

$$R \cdot \sin(\delta) = (1 - 2F') \cdot R \cdot \sin(\psi/2) \quad (22)$$

Solving for δ results in

$$\delta = \sin^{-1}[(1 - 2F') \cdot \sin(\psi/2)] \quad (23)$$

Summing the angles in Fig. 9, for $\omega \le (\psi/2)$, results in

$$\omega = \psi - \psi/2 - \delta \quad (24)$$

If F' in Eq. (23) is replaced with F_c, δ becomes negative for $F_c > 0.5$. As a result, Eq. (24) is correct if δ is substituted for using F_c in Eq. (23) instead of F'. The result is

$$\omega = \psi/2 - \sin^{-1}[1 - 2F_c) \cdot \sin(\psi/2)] \quad (25)$$

Substitution of ω into (20) results in

$$F_m = 1/2 - (1/\psi) \cdot \{\sin^{-1}[(1 - 2F_c) \cdot \sin(\psi/2)]\} \quad (26)$$

Solving for F_c yields

$$F_c = 0.5 \cdot \{1 - \text{in}[\psi \cdot (0.5 - F_m)]/\sin(\psi/2)\} \quad (27)$$

Derivation of Arc Displacement, D_a. Line segments, $\underline{I\text{-}K}$ and $\underline{J\text{-}M}$ are normal to the chord. The distance between the chord and the arc at the arbitrary point, K, is the arc displacement, D_a. By construction and for all $F_c \le 0.5$, I-D equals D-J, I-K equals J-M and S1-I equals J-S2. Further, segment I-J equals segment $\underline{S1\text{-}S2}$ less the segment sum, (S1-I + J-S2). The resulting expression for $\underline{I\text{-}J}$ is (for F' as defined in Eq. (17)):

$$\underline{I\text{-}J} = (1 - 2F') \cdot 2R \cdot \sin(\psi/2) \quad (28)$$

Because K-L equals $\underline{I\text{-}J}/2$ and $R - \sin(\delta)$, the following can be written:

$$R \cdot \sin(\delta) = (1 - 2F') \cdot R \cdot \sin(\psi/2) \quad (29)$$

Solving for the angle, δ, gives

$$\delta = \sin^{-1}[(1 - 2F') \cdot \sin(\psi/2)] \quad (30)$$

The arc displacement, D_a, is equal to $\underline{I\text{-}K}$ or $\underline{J\text{-}M}$; and $\underline{I\text{-}K}$ is equal to $\underline{H\text{-}K}$ less $\underline{C\text{-}D}$ (or $\underline{H\text{-}I}$): Because $\underline{H\text{-}K} = R \cdot \cos(\omega)$ and $\underline{C\text{-}D} = R \cdot \cos(\psi/2)$, the equation for D_a is

$$D_a = R \cdot [\cos(\delta) - \cos(\psi/2)] \quad (31)$$

In terms of known variables and in oilfield units, the radius of a circular arc is $(180 \cdot \Delta MD)/(\pi \cdot \psi)$. Substituting δ and R in Eq. (31) gives

$$D_a = [(180 \cdot \Delta MD)/(\pi \cdot \psi)] \cdot \{\cos[\sin^{-}[(1 - 2F_c) \cdot \sin(\psi/2)]] - \cos(\psi/2)\} \quad (32)$$

APPENDIX C

Derivation of Direction Cosines for the Arc displacement, D_a

The change in vertical depth between S1 and B is $\underline{S1 - F}$ (refer to Fig. 8). The change in the X and Y directions between S1 and B is found by multiplying $\underline{G\text{-}A}$, by $\sin(\theta_1)$ and $\cos(\theta_1)$, respectively. These relative displacements can be expressed as

$$\Delta X_b = R \cdot \tan(\psi/2) \cdot \sin(\phi_1) \cdot \cos(\theta_1) \quad (33)$$

$$\Delta Y_b = R \cdot \tan(\psi/2) \cdot \sin(\phi_1) \cdot \sin(\theta_1) \quad (34)$$

$$\Delta Z_b = R \cdot \tan(\psi/2) \cdot \cos(\phi_1) \quad (35)$$

The inclination of the chord is the angle whose cosine is equal to $\underline{S1\text{-}G}$ divided by $\underline{S1\text{-}S2}$. After simplification of terms, this expression is

$$INC_{chord} = \cos^{-1}\{[(\phi_1 + \phi_2)/2]\cos(\psi/2)\} \quad (36)$$

The vertical displacement between S1 and D is equal to $\underline{S1\text{-}D}$ multiplied by the cosine of the inclination of the chord. After simplification of terms, this equation is

$$\Delta Z_d = R \cdot \tan(\psi/2) \cdot [\cos(\phi_1) + \cos(\phi_2)]/2 \quad (37)$$

Because D is the midpoint of the chord of the circular arc between S1 and D, the change in X and Y displacements between S1 and D are the sum of one-half of segment $\underline{G\text{-}A}$ times the sine an cosine, respectively, of AZM1 and one-half of segment $\underline{A\text{-}S2}$ times the sine and cosine, respectively, of AZM2. After simplification, the equations are

$$\Delta X_d = 1/2 \cdot R \cdot \tan(\psi/2) \cdot [\sin(\phi_1)\sin(\theta_1) + \sin(\phi_2)\sin(\theta_2)] \quad (38)$$

$$\Delta Y_d = 1/2 \cdot R \cdot \tan(\psi/2) \cdot [\sin(\phi_1)\cos(\theta_1) + \sin(\phi_2)\cos(\theta_2)] \quad (39)$$

The coordinate displacements of $\underline{D\text{-}B}$ are computed by subtracting the coordinates for D from those for B. The equations are

$$D_x = R \cdot \tan(\psi/2) \cdot [\sin(\phi_1)\sin(\theta_1) - \sin(\phi_2)\sin(\theta_2)]/2 \quad (40)$$

$$D_y = R \cdot \tan(\psi/2) \cdot [\sin(\phi_1)\cos(\theta_1) - \sin(\phi_2)\cos(\theta_2)]/2 \quad (42)$$

$$D_z = R \cdot \tan(\psi/2) \cdot \cos(\phi_1) - [\cos(\phi_2)]/2$$

Let ρ^* equal the magnitude of $\underline{D\text{-}B}$, and ρ^* is

$$\rho^* = \sqrt{(D_x)^2 + (D_y)^2 + (D_z)^2} \quad (43)$$

After substitution and reduction, this gives

$$\rho^* = 0.5R \cdot \tan(\psi/2) \cdot \sqrt{2 \cdot [1 - \sin(\phi_1)\sin(\phi_2)\cos|\Delta\theta| - \cos(\phi_1)\cos(\phi_2)]} \quad (44)$$

Any displacements between the chord and the arc is parallel to $\underline{D\text{-}B}$. The direction cosines for any set of coordinate displacements between the chord and the arc are expressed by dividing each displacement (Eqs. (40)–(42)) by the magnitude of $\underline{D\text{-}B}$. If ρ is defined as $\rho^*/0.5R \cdot \tan(\psi/2)$, the result is

$$\rho = \sqrt{2 \cdot [1 - \sin(\phi_1)\sin(\phi_2)\cos|\Delta\theta| - \cos(\phi_1)\cos(\phi_2)]} \quad (45)$$

The direction cosines for all line segments parallel to D-B can be expressed

$$\alpha = [\sin(\phi_1)\sin(\theta_1) - \sin(\phi_2)\sin(\theta_2)]/\rho \quad (46)$$

$$\beta = [\sin(\phi_1)\cos(\theta_1) - \sin(\phi_2)\cos(\theta_2)]/\rho \quad (47)$$

$$\gamma = [\cos(\phi_1) - \cos(\phi_2)]/\rho \quad (48)$$

APPENDIX D

Derivation of Interpolation Equations

Reference is now to Fig. 8 for derivation of equations for interpolation by the sectional method. Let point K be at the measured depth at which interpolation is desired from the circular arc. Let the change in measured depth between S1 and K divided by the total change in measured depth between S1 and S2 be the interpolation ratio, F_m. Let the distance along the chord between S1 and I divided by the total chord length between S1 and S2 be the interpolation ratio, F_c. Line segments, I-K and J-M are normal to the chord. The distance between the chord and the arc at the arbitrary point, K, is the Arc Displacement, D_a. Note, for I-D = D-J, that I-K = J-M. The ratios, F_m and F_c, are related by the following equation (see Appendix B for derivation):

$$F_c = 0.5 \cdot \{1 - \sin[\psi \cdot (0.5 - F_m)] / \sin(\psi/2)\} \tag{27}$$

The distance from the chord to the arc is computed with the following formula (see Appendix B for derivation) for the arc displacement , D_a:

$$D_a = [180 \cdot \Delta MD / (\pi \cdot \psi)] \cdot \{\cos[\sin - 1[(1 - 2F_c) \cdot \sin(\psi/2)]] - \cos(\psi/2)\} \tag{32}$$

Let α, β and γ be the direction cosines (X, Y and Z = axes, respectively) for the line segment D-B (see Appendix C for derivation). Also, as in Appendix C, ρ is defined as

$$\rho = \sqrt{2 \cdot [1 - \text{in}(\phi_1)\sin(\phi_2)\cos|\Delta\theta| - \cos(\phi_1)\cos(\phi_2)]}$$

The equations for the direction cosines are

$$\alpha = [\sin(\phi_1)\sin(\theta_1) - \sin(\phi_2)\sin(\theta_2)] / \rho \tag{46}$$

$$\beta = [\sin(\phi_1)\cos(\theta_1) - \sin(\phi_2)\cos(\theta_2)] / \rho \tag{47}$$

$$\gamma = [\cos(\phi_1) - \cos(\phi_2)] / \rho \tag{48}$$

Let ΔX_t, ΔY_t and ΔZ_t be displacements between stations S1 and S2; and let ΔX_i, ΔY_i and ΔZ_i be displacements between S1 and point K

$$\Delta X_i = F_c \cdot \Delta X_t + D_a \cdot \alpha \tag{49}$$

$$\Delta Y_i = F_c \cdot \Delta Y_t + D_a \cdot \beta \tag{50}$$

$$\Delta Z_i = F_c \cdot \Delta Z_t + D_a \cdot \gamma \tag{51}$$

Interpolated Azimuth and Inclination. The inclination at point K is ϕ_2 in Eq. (11) after ΔZ_i is substituted for ΔTVD, ϕ_1 for ϕ_2, and $\psi_i = F_m \cdot \psi$

$$\phi_i = \cos^{-1}\{[\Delta Z_i / R \cdot \tan(\psi_i/2)] - \cos(\phi_1)\} \tag{52}$$

The change in azimuth can be found by substituting the foregoing variables into Eq. (6) and solving for $\Delta\theta$

$$\Delta\theta = \cos^{-1}\{[2 \cdot \cos(\psi_i/2) - 1 - \cos(\phi_1)\cos(\phi_i)] / [\sin(\phi_1)\sin(\phi_i)]\} \tag{53}$$

Equation (53) provides only the magnitude of the azimuth change. The direction of change is assumed to be continuous from the first station to the second and can easily be computed by

$$(\theta_2 - \theta_1)/\Delta\theta$$

Then the interpolated azimuth at the intermediate station is

$$\theta_i = \theta_1 \pm \Delta\theta \tag{54}$$